COMPUTABLE GENERAL EQUILIBRIUM MODEL FOR THE REPUBLIC OF MOLDOVA

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This article is devoted to the construction of a small Computable General Equilibrium model for the Republic of Moldova using Social Accounting Matrix build on the statistical data, 2015 as the base year. The model 1-2-3 formed a base for constructing Computable General Equilibrium Model. Using data from SAM, Lagrange method, optimization techniques, the model has been adjusted to the real situation from the Republic of Moldova. It means that all coefficients of the behavioral functions have been determined. Then, the obtained model has been applied to assess the economic evolution of Moldova based on diverse exogenous scenarios. Actuality of this research consists in elaborating new techniques – SAM (for the first time in the Republic of Moldova) for economic analysis and simulation calculus using the General Equilibrium Model.

Keywords: *Social Accounting Matrix, Computable General Equilibrium Model, 1-2-3 model, Lagrange method, optimization methods, simulation calculus.*

Acest articol este consacrat construirii unui model mic de Calcul de Echilibru General folosind Matricea de Calcul Social pentru Republica Moldova elaborată în baza datelor statistice ale anului de bază, anul 2015. Modelul de tip 1-2-3 a stat la baza construirii Modelului de Calcul de Echilibru General. Pornind de la datele Matricei de Calcul Social, folosind metoda multiplicatorilor Lagrange, tehnicile de optimizare, modelul a fost calibrat pentru situaţia reală din Republica Moldova. Ceea ce înseamnă că s-au calculat toţi parametrii funcţiilor comportamentale implicate în model. Apoi modelul obţinut s-a aplicat la evaluarea evoluţiei economice a Republicii Moldova în baza diverselor scenarii exogene. Actualitatea acestei cercetări constă în construirea (pentru prima dată) în Republica Moldova a unui instrumentar nou pentru efectuarea analizei economice şi în baza ei calculelor de simulare cu ajutorul Modelului de Calcul de Echilibru General.

Cuvinte-cheie: *Matricea de Calcul Social, Modelul de Calcul de Echilibru General, Modelul 1-2-3, metoda multiplicatorilor Lagrange, metode de optimizare, calcule de simulare.*

В настоящей статье была построена небольшая Вычислительная Модель Общего Равновесия, используя Матрицу Социальных Расчетов для Республики Молдова, разработанную на основе статистических данных базового 2015 года. Модель типа 1-2-3 составляет основу конструирования Вычислительной Модели Общего Равновесия. Отталкиваясь от данных Матрицы Социальных Расчетов, используя метод множителей Лагранжа, оптимизационные техники, была произведена калибровка модели, основываясь на реальной экономической ситуации Республики Молдова. Это означает, что были рассчитаны все параметры поведенческих функций, составляющих модель. Далее, окончательная модель была использована для оценки экономического развития республики Молдова, используя различные экзогенные сценарии. Актуальность настоящего исследования заключается в том, что, впервые в Республике Молдова, была построена Матрица Социальных Расчетов, на основе которой была разработана Вычислительная Модель Общего Равновесия и проведены сценарные расчеты.

Ключевые слова: *Матрица Социальных Расчетов, Вычислительная Модель Общего Равновесия, Модель 1-2-3, метод множителей Лагранжа, методы оптимизации, сценарные расчеты.*

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Introduction

The main goal of this article is the construction of a small Computable General Equilibrium model for the Republic of Moldova. The static version of the model is examined. Economy as a hole is assumed to be perfectly competitive, meaning zero profit is earned in the economy. Small country assumption ensures that Moldova is a price taker on the world market, so that Moldova's import and export decisions do not affect international prices. The model belongs to the 1-2-3 class of CGE models. The basic model refers to one country with two producing sectors and three goods. No factor markets are considered. The two commodities that the country produces are an export good E , which is sold to foreigners and is not demanded domestically, and a domestic good D , which is only sold domestically. The third good is an import M , which is not produced domestically. There is one consumer who receives all income. Our country is small in world markets, so facing fixed world prices for exports and imports.

This assumption facilitates the tractability of results, and makes the model flexible enough to incorporate features specific to the Republic of Moldova's economy. Such a model was studied earlier in [1-2]. Present version of the model is calibrated dealing with construction of the Social Accounting Matrix for the base year, 2015 year, using National Accounts data elaborated by National Bureau of Statistics [3].

Scientific approach

Scientific approach is based on the general equilibrium modern theory. Debreu [4], Arrow and Hahn [5] formalized the concept and rigorously proved the existence of equilibrium in the economy where agents make independent decisions. In the competitive Arrow-Debreu equilibrium, demand and supply decisions depend only on the relative prices.

The economy is characterized by private ownership, consumers own the resources and receive all revenues from production. However, it is possible to construct a GE model in a way that equilibrium would be achieved even without the model satisfying the Walras's Law [6]. Another important concept introduced by Walras was the excess demand function, the difference between total demand and the sum of total resources and total supply, at a given price.

The mathematical problem of proving the existence of equilibrium reduces to finding a set of prices P , which would make corresponding excess demand zero for every commodity. The problem was solved by the joint effort of Arrow and Debreu [7] with the help of Brouwer's and Kakutani [8] fixed point theorems.

Shoven and Whalley [9-10] proved the existence of the general equilibrium solution in the presence of ad valorem producer and consumer taxes. It contributed for the development of empirical applications of the GE theory. Shoven [10] includes taxes and government into the Arrow-Debreu framework. The difference from the classic Arrow-Debreu GE model is that now consumer demands and incomes depend not only on prices, but on demand and supply decisions of other consumers and producers in the economy.

The computational general equilibrium model of international trade is also demonstrated by Shoven and Whalley [11], which provides an alternative proof of existence of a competitive equilibrium with international markets with tariffs. Later research has been concentrated on the issues of stability and uniqueness of the competitive equilibrium as well on implications of dynamics [12-15].

The purpose and scientific basis of the research

Main objective of this research is referred to construction of the Calculated General Equilibrium Model for the Republic of Moldova based on the elaboration of the Social Accounting Matrix, studied in [16-17]. Using statistical data for year 2015, as the base year, the Social Accounting Matrix for the Republic of Moldova was created. This matrix was used to calibrate behavioral parameters for the Calculated General Equilibrium Model. After model calibrating diverse scenarios were realized and analyzed. Theoretical background of this research is based on the optimization methods, Lagrange method, solution of the non linear system of equations, application of the Solver procedure for its solution. Four sectors model was examined: the private sector represented by households, public sector represented by the government, production sector represented by the firms and foreign sector represented by rest of the world. Macro version of the Calculated General Equilibrium Model elaborated in [18] has been studied and adopted to the economic realities of the Republic of Moldova and the same notation has been used.

1. Private Sector (Households)

Private sector income is constituted by:

$$Y = PkKS + Pl(LS - Unemp) + GHtrf + R \cdot WHtrf + CPI \cdot HHtrf, \quad \text{here} \tag{1}$$

KS and LS are the total capital and labor supplies, $Unemp$ is the level of unemployment, $GHtrf$ represent transfers of unemployment benefits, $WHtrf$ and $HHtrf$ are remittances and inter-household transfers accordingly, and Pk and Pl are capital rental rate and nominal wage.

From its income a household pays income tax $ty \cdot Y$ and social contribution $HGtrf$ to the government and inter-household transfers $HHtrf$. It also transfers a part of its income to the rest of the world $HWtrf$. In the case when all capital in the model is assumed to be owned by the households, this variable represents transfers, such as the return on foreign owned capital, income from foreign labor and other transfers abroad. A fixed share of the remaining income is saved according to (2).

$$Sh = mps \cdot (Y - ty \cdot Y - R \cdot HWtrf - CPI \cdot HGtrf - CPI \cdot HHtrf) \quad (2)$$

Here, mps is a parameter of marginal propensity to save. Real social contribution $GHtrf$ is fixed, while nominal value changes in respect to inflation. Transfers to the rest of the world are expressed in foreign currency so multiplied by the nominal exchange rate R.

So, the net income or household's budget available for consumption B is defined as

$$B = Y - ty \cdot Y - Sh - R \cdot HWtrf - CPI \cdot HGtrf - CPI \cdot HHtrf \quad (3)$$

The last remaining element of a household's spending is the final consumption. The Stone-Geary household utility function will be examined

$$U(C) = (C - \mu)^{\alpha h}. \quad (4)$$

Here μ is in the subsistence consumption level, which a household has to obtain before any other consumption. All remaining income is spent on consumption above that level, according to budget share. The subsistence level can determine how much consumption is determined by demand or supply. Larger (smaller) shares make demand less (more) responsive to variations in price or income.

Thus, a household solves the problem of maximizing Stone-Geary utility function (4) by choosing consumption level C subject to budget constraint (6):

$$\max_C \; U(C), \quad (5)$$

$$\text{subject to} \quad B = (1 + tc)PC \cdot, \text{ where } C > \mu \geq 0, \quad (6)$$

tc is the tax on final consumption.

Applying Lagrange method, from the first order conditions, the solution to this problem Linear Expenditure System (LES) determines household's consumption demand

$$C = \mu + \frac{\alpha h (B - (1 + tc) \cdot P\mu)}{(1 + tc)P} \quad (7)$$

In order to calibrate parameters αh and μ: Frisch parameter φ (expenditure elasticity of the marginal utility of expenditure); and household's income elasticity of demand for commodities $elasH$ have to be selected from outside.

2. Public Sector (Government)

It is assumed that a government maximizes Cobb-Douglas utility function subject to budget constraints. Government collects all taxes and social contributions, pays unemployment benefits, and saves a fixed amount of its revenues. Equation (8) defines the total government income as the sum of revenues from indirect taxes on final consumption tc, capital tax tk, taxes on intermediate consumption tic import tariffs tm and export duties te, also revenues from personal income tax ty,social contributions $GHtrf$ and transfers from abroad $WHtrf$.

$$CB = (tc \cdot PC + tk \cdot Pk \cdot K + tic \cdot P \cdot Xt) + (tm \cdot R \cdot Pmw + te \cdot R \cdot Pe \cdot E) + y \cdot Y + CPI \cdot HGtrf + \\ + R \cdot WGtrf \quad (8)$$

Equation (9) states that unemployment benefits, paid by the government, are a fraction hub, of the nominal wage: $GHtrf = hub \cdot Pl \cdot Unemp$ (9)

The government's final demand G is assumed to be endogenous, where saving Sg is exogenous. Consequently, the government's demand for commodities derived from maximizing the Cobb-Douglas utility function (10) subject to budget constraints (11):

$$\max_G \; G^{\alpha g} \quad (10)$$

$$\text{Subject to } CB - GHtrf - CPI \cdot Sg - R \cdot GWtrf = PG \quad (11)$$

The left-hand side of equation (11) represents the government's expenditure budget including the service of foreign debt $GWtrf$. From the first order conditions, using Lagrange multipliers method, the government's demand for commodities is obtained $G = \frac{\alpha g\left(GB - GHtrf - CPI \cdot Sg - R \cdot GWtrf\right)}{P}$ (12)

The conventional way to implement government behavior in CGE models would be to assume fixed real expenditure, allowing the nominal expenditure to vary with inflation and savings to balance the government budget (negative savings meaning budget deficit, positive budget surplus).

3. Production Sector (Firm)

There is one aggregate firm which observes prices and makes output decisions in order to maximize profit subject to technology constraints. This firm produces only one commodity using capital, labor and intermediates as inputs. Capital and Labor are combined into composite Value added goods using the CES function and Intermediates are supposed to be given.

Formally, aggregate firm chooses capital and labor by minimizing the total cost of production, TC :

$$TC = (1 + tk)Pk \cdot K + Pl \cdot L + (1 + tic) \cdot P \cdot Xt . \quad (13)$$

Subject to technological constraint $Xt = \alpha f \left(\gamma fk \cdot K^{(\sigma fk - 1)/fk} + (1 - \gamma fl)L_1^{(\sigma fl - 1)/\sigma fl}\right)^{\frac{\sigma f}{\sigma f - 1}}$, (14)

$\sigma fl, \sigma fk$ are the elasticity of substitution between labor and capital, αf is the efficiency parameter and γfk, γfl are the share parameters such as $\gamma fk + \gamma f = 1$. Xt is total domestic output. Using Lagrange multipliers method, from the first order conditions firms' labor and capital demand equations are given as:

$$L = \left(\frac{1 - \gamma fl}{Pl}\right)^{\sigma fl_i} \left(\frac{X_t}{\alpha fl}\right)\left(\gamma fl^{\sigma fl}\left[(1 + tk)Pk\right]^{1-\sigma fl} + (1 - \gamma fl)^{\sigma fl} Pl^{1-\sigma fl}\right)^{\frac{\sigma fl}{1-\sigma fl}} \quad (15)$$

$$K = \left(\frac{\gamma fk}{(1 + tk)Pk}\right)^{\sigma fk} \left(\frac{Xt}{\alpha fk}\right)\left(\gamma fk^{\sigma fk}\left[(1 + tk)Pk\right]^{1-\sigma fk} + (1 - \gamma fk)^{\sigma fk} Pl^{1-\sigma fk}\right)^{\frac{\sigma fk}{1-\sigma fk}} \quad (16)$$

In such a way, the aggregate firm chooses the production factors to produce a unit of its output in the most cost-effective way; however, it has not been known how much output this aggregate firm should produce. It is supposed that firm chooses its output level in order to maximize total profit. Because the perfectly competitive economy and constant returns to scale production function are considered, income is equal to zero, hence firm's zero profit condition equation (17) is used for output:

$$Pt \cdot Xt = (1 + tk) \cdot Pk \cdot K + Pl \cdot L + (1 + tic) \cdot P \cdot Xt \cdot \quad (17)$$

***4.* Savings-Investments**

The model is savings driven in the sense that aggregate investment is the sum of sartorial savings components. Equation (18) depicts total savings in the economy as the sum of private savings Sh defined by the fixed savings rate, government savings Sg and foreign savings Sf are determined exogenously:

$$S = Sh + CPI \cdot Sg + R \cdot Sf \quad (18)$$

Because the model is static, investment demand is defined as a constant fraction αi of the total savings S and it varies in accordance with commodity price P. In mathematic terms, investment demand is derived by maximizing the utility function of an auxiliary agent: $\max_I \; I^{\alpha i}$ (19)

Subject to savings constraint $S = P \cdot I$ (20)

The solution to this optimization problem will be the investment demand equations:

$I = \frac{\alpha i \cdot S}{P}$ (21), parameter αi can be calibrated from the equation (21) using known values of the S, P, I .

Foreign sector (Rest of the world)

The small country assumption implies that world prices are treated as exogenous, in consequence, changes in the Republic of Moldova's import demand or export supply don`t have any impact on the world economy. Therefore, the domestic import and export prices can be represented as follows:

$$Pm = (1 + tm)R \cdot Pmw \quad (22)$$

$$Pe = Pew \cdot R / (1 + te). \quad (23)$$

Where tm and te are import tariffs and export duties; Pe and Pm are prices received by domestic producers for selling their output on the foreign or domestic markets; Pm represents domestic price of imported goods; Pmw and Pew are exogenous world imports and exports prices correspondingly, and R is the nominal exchange rate.

Export and import followed the treatment adopted in many CGE models. Farmington's assumption [12] ensures that imports and domestically produced goods are not perfect substitutes meaning that not all products can be produced domestically. The aggregate firm chooses between selling output on the domestic market or exporting it depending on the relative prices and transformation elastic ties. Such treatment allows exporting and importing the same good simultaneously.

Total domestic output Xt of a representative firm is either sold on the domestic market Xd or exported E. The transformation takes place within CET (constant elasticity of transformation) function; therefore the optimal combination of exports and domestic sales of an aggregate good depends only on the relative (domestic to foreign) price of that good. To determine export and domestic production, the firm solve the following problem: Maximize *total revenue-TR*

$$TR = Pe \cdot E + Pd \cdot Xd \tag{24}$$

Subject to the constant elasticity of transformation technology

$$Xt = ae\left(\gamma e \cdot M^{\frac{\sigma e-1}{\sigma e}} + (1-\gamma e)Xd^{\frac{\sigma e-1}{\sigma e}}\right)^{\frac{\sigma e-1}{\sigma e}} \tag{25}$$

Where ae, γe and σe are transformation efficiency parameters, CET distribution parameter and transformation elasticity.

Applying Lagrange multipliers method, the solution to this problem of domestic and export sales of domestic output will be obtained:

$$Xd = \left(\frac{1-\gamma e}{Pd}\right)^{\sigma e}\left(\frac{Xt}{ae}\right)\left(\gamma e^{\sigma e} Pe^{1-\sigma e} + (1-\gamma e)^{\sigma e} Pd^{1-\sigma e}\right)^{\frac{\sigma e}{1-\sigma e}} \tag{26}$$

$$E = \left(\frac{\gamma e}{Pe}\right)^{\sigma e_i}\left(\frac{Xt}{ae}\right)\left(\gamma e^{\sigma e} Pe^{1-\sigma e} + (1-\gamma e)^{\sigma e} Pd^{1-\sigma e}\right)^{\frac{\sigma e}{1-\sigma e}} \tag{27}$$

Since there is zero profit condition, from which directly follows:

$$Pt \cdot Xt = Pe \cdot E + Pd \cdot Xd \tag{28}$$

Imports are combined with domestic output in the CES function to produce composite commodity X. The optimal mixture depends only on the relative prices. Demand for imports and locally produced goods, derived from minimizing the total cost subject to CES technology constraint, thus the following optimization problem:

Minimize *total cost - TC*

$$TC = Pm \cdot M + Pd \cdot Xd \tag{29}$$

Subject to $$X = am\left(\gamma m \cdot M^{\frac{\sigma m-1}{\sigma m}} + (1-\gamma m)\cdot Xd^{\frac{\sigma m-1}{\sigma m}}\right)^{\frac{\sigma m-1}{\sigma m}}. \tag{30}$$

am, γm and σm are respectively, substitution efficiency parameter, CES distribution parameter and elasticity of substitution between imported and local goods.

Again, using Lagrange multipliers method, from the first order conditions the demands for import and domestic goods are obtained:

$$Xd = \left(\frac{1-\gamma m}{Pd}\right)^{\sigma e}\left(\frac{Xt}{\alpha m}\right)\left(\gamma m^{\sigma m} \cdot Pe^{1-\sigma m} + (1-\gamma m)^{\sigma m} \cdot Pd^{1-\sigma m}\right)^{\frac{\sigma m}{1-\sigma m}} \tag{31}$$

$$M = \left(\frac{\gamma m}{Pm}\right)^{\sigma e}\left(\frac{X}{am}\right)\left(\gamma m^{\sigma m} \cdot Pe^{1-\sigma m} + (1-\gamma m)^{\sigma m} \cdot Pd^{1-\sigma m}\right)^{\frac{\sigma m}{1-\sigma m}} \tag{32}$$

The price P of composite commodity X is defined using zero profit condition:

$$P \cdot X = Pm \cdot M + Pd \cdot Xd \quad (33)$$

Several parameters have to be imposed from outside the model, which will allow calibrating the rest. Specifically, assuming the negative (positive) values for the substitution elasticity σe (σm) ae and γe, (am and γm) can be derived by solving systems of two nonlinear equations, values of the variables M, X, Xd, Xt, Pd, Pe will be selected from the SAM. The CES (CET) functions are well behaved, namely if the elasticity of substitution is negative the function will be concave and if positive it will be convex. This property ensures that first order conditions for revenue maximization (cost minimization) produce the desired maximum (minimum) solution.

The balance of payment equilibrium condition (34) finishes the description of the foreign sector. With Being fixed Sf and $HWtrf$, equilibrium is achieved through the exchange rate variations, which adjust the levels of export and import via the change of its relative prices [19].

$$Pmw \cdot M + HWtrf - GWtrf = Pew \cdot E + Sf + WGtrf + WHtrf \quad (34)$$

Production factors (Labor and Capital)

In the basic CGEM specification, total capital stock and labor supply are fixed exogenously, and factor markets clear according to equation (35).

$$LS = L + Unemp, \; KS = K. \quad (35)$$

Commodities` market clearing, CPI and balanced model

The last macroeconomic closure corresponds to commodities' market clearing equilibrium conditions. Equation (36) states that total domestic supply (including imported goods) must equal total demand.

$$X = Xt + G + C + I \quad (36)$$

The consumer price index CPI is Laspeyres and is defined as:

$$CPI = \frac{(1 + tc^{t}) \cdot P^{t} \cdot C^{0}}{(1 + tc^{0}) \cdot P^{0} \cdot C^{0}}, \; t = 0,1 \quad (37)$$

Equation (37) concludes the description of the small CGE model for the Republic of Moldova. The algebraic form of the model is a system of nonlinear equations, solution to which determined the equilibrium state. For such a system to have a unique solution, the number of independent variables should be equal to the number of independent equations. Carefully counting all variables and equations results in:

$NE = 15 + 11$ - Number of equations

$NV = 15 + 21$ - Number of variables

Considering ten variables ($GHtrfo, HWtrf, HGtrf, HHtrf, WHtrf, GWtrf, Sf, Sg, KS, LS$) at their initial level, there is $NV = NE = 26$. The heterogeneity of some of these variables has already been discussed; others had to be made exogenous in real terms to balance the model, however nominally they can change. Furthermore, since the model satisfies Walras's law there is only $15 + 10$ independent equations. By dropping one of the equilibrium conditions from the model (in this case it is the labor market clearing condition (35), and fixing nominal wage Pl at its initial level (thus making it to be the numeraire) we again have the number of variables equal to the number of equations.

Constant Elasticity of Substitution Function (Example of calibration)

In this calibration all calculations are done assuming taxes to be zero. This simplification by no means changes the nature of equations and in order to include tax on production factors, one would simply need to replace Pl, Pk to $(1 + tl)Pl, (1 + tk)Pk$. There is one aggregated firm in the economy and it produces one aggregated commodity. Thus, for the representative firm, which faces CES production technology, demand for factors of production (Ls, K), can be derived in the following way.

$$\min_{L_s, K} TC = Pl \cdot Ls + Pk \cdot K$$

Subject to $Xt = af\left(\gamma fl \cdot Ls^{-\rho} + \gamma fk \cdot K^{-\rho}\right)^{-\frac{1}{\rho}}$,

TC is total cost of production, af is the efficiency parameter, Ls, K and Pl, Pk represent demand

and price for labor and capital production factors, $\gamma fl , \gamma fk$ are the share parameter such as $\gamma fl + \lambda fk = 1$, and $\rho = \frac{1-\sigma f}{\sigma f}$ where σf is the elasticity of substitution. This optimization problem will have the following Lagrangian: $L = Pl \cdot Ls + Pk \cdot K_k - \lambda\left(af\left(\gamma fl \cdot Ls^{-\rho} + \gamma fl \cdot K^{-\rho}\right)^{-\frac{1}{\rho}} - Xt\right)$

Taking the first order conditions and making it equal to zero we get:

$$\frac{\partial L}{\partial Ls} = Pl - \lambda \cdot af \cdot \gamma fl \cdot Ls^{-(1+\rho)}\left(\left(\gamma fl \cdot Ls^{-\rho} + \gamma fk \cdot K^{-\rho}\right)\right)^{-\left(1+\frac{1}{\rho}\right)} = 0$$

$$\frac{\partial L}{\partial K} = Pk - \lambda \cdot af \cdot \gamma fk \cdot K^{-(1+\rho)}\left(\left(\gamma fl \cdot L^{-\rho} + \gamma fk \cdot K^{-\rho}\right)\right)^{-\left(1+\frac{1}{\rho}\right)} = 0$$

$$\frac{\partial L}{\partial \lambda} = af\left(\gamma fl \cdot Ls^{-\rho} + \gamma fk \cdot K^{-\rho}\right)^{\frac{1}{\rho}} - Xt = 0$$

Dividing first order conditions for Ls by the first order conditions for K is obtained the following expression :

$$\frac{\partial L / \partial Ls}{\partial L / \partial K} = -\frac{Pl_k}{Pk_s} + \frac{\gamma fl}{\gamma fk}\left(\frac{Ls}{K}\right)^{-(1+\rho)} = 0$$

From what follows $Ls = K\left(\frac{Pl}{Pk} \cdot \frac{\gamma fk}{\gamma fl}\right)^{1/(-(1+\rho))}$

Now substitute Ls into expression for Xt to get

$$Xt = af \cdot K\left(\frac{\gamma fk}{Pk}\right)^{\frac{1}{-(1+\rho)}}\left(\gamma fl^{1/(1+\rho)} \cdot Pl^{\rho/(1+\rho)} + \gamma fk^{1/(1+\rho} \cdot Pk^{\rho/(1+\rho)}\right)^{-\frac{1}{\rho}}$$

and after some rearrangements, the aggregate firm's demand for production factors are:

$$Ls = \left(\frac{\gamma fl}{Pl}\right)^{1/(1+\rho)}\left(\frac{Xt}{afl}\right)\left(\gamma fl^{1/(1+\rho)} \cdot Pl^{\rho/(1+\rho)} + \gamma fk^{1/(1+\rho)} \cdot Pk^{\rho/(1+\rho)}\right)^{\frac{1}{\rho}}$$

$$K = \left(\frac{\gamma fk}{Pk}\right)^{1/(1+\rho)}\left(\frac{Xt}{afk}\right)\left(\gamma fl^{1/(1+\rho)} \cdot Pl^{\rho/(1+\rho)} + \gamma fk^{1/(1+\rho)} \cdot Pk^{\rho/(1+\rho)}\right)^{\frac{1}{\rho}}$$

Calibration

Assuming substitution elasticity σf is given from outside of the model, then af , γfk , γfl can be calculated.

$$\frac{\gamma fl}{\gamma fk} = \frac{Pl}{Pk}\left(\frac{Fl}{Fk}\right)^{1+\rho}$$

From $\gamma fl + \gamma fk = 1$ we can find $\gamma fk = 1 - \gamma fl$ and substitute it into previous equality

$$\frac{\gamma f_l}{1-\gamma fl} = \frac{Pl}{Pk}\left(\frac{L}{K}\right)^{1+\rho} , \qquad \gamma fk = \frac{1}{1 + \frac{Pl}{Pk}\left(\frac{K}{L}\right)^{1+\rho}}$$ and $\gamma fl = 1 - \gamma fk$

Once we have σf and γf_k, remembering that $\rho = \frac{1-\sigma f}{\sigma f}$ af can be derived from the following equation:

$af = Xt\left(\sum_{k=1}^{m} \gamma f_k F_k^{-\rho}\right)^{\frac{1}{\rho}}$, Solving optimization problem for base year given Xt , Ls , K , parameters afl , afk have been determined. The same procedure can be applied to obtain the rest of parameters.

1. Variables and Parameters of the model

Nominal variables

CPI	–	Consumer price index	B	–	Household income spent on consumption
P	–	Price of composite commodities	GB	–	Government total budget
Pt	–	Produce price	Y	–	Household total income
Pd	–	Price of domestic output on the local market	R	–	Exchange rate
Pk	–	Price of capital	S	–	Total savings
Pl	–	Price of labor	Sh	–	Household savings
Pe	–	Price of export	GHtrf	–	Transfers from government to household
Pm	–	Price of import		–	
Pew	–	World price of export (exogenous)		–	
Pmw	–	World price of import (exogenous			

Auxiliary variables

U – Household\`utility; TC – Total cost; TR – Total revenue

Parameters

μ	–	Household minimum consumption	afl	–	Firm CES efficiency parameter (labor)
αh	–	Power parameter in the household S-G utility function	afk	–	Firm CES efficiency parameter (capital)
αg	–	Power parameter in the government C-D utility function	am	–	Armington CES efficiency parameter
αi	–	Power parameter in the investment C-D utility function	ae		CET efficiency parameter
mps	–	Household marginal propensity to save	tc	–	Tax rate on final consumption
I	–	Investment demand	tk	–	Tax rate on capital
γfl	–	Firm CES distribution parameter (labour)	ty	–	Tax rate on household income
γfk	–	Firm CES distribution parameter (capital)	tm	–	Tax rate on import
γe	–	CET distribution parameter	te	–	Export duty rate
γm	–	Armington CES distribution parameter	ti	–	Tax rate on intermediate consumption

Results of own research

To solve the Calculated General Equilibrium Model presented earlier it is necessary to give values for all exogenous variables of the model, to determine some calculated parameters, to calibrate all coefficients for behavioral functions obtained throw optimization problems solving.

Table 1

Exogenous parameters

hub	ρ	σf	σm	Σe	elasH	Φ
0,187	-0,1	1,1	3,0	-2,0	1,0	-1,1

Table 2

Calculated parameters, year 2015

tc	tk	ty	Tm	Te	ti	mps	infl	ω	R	P
0,1589	0,1928	0,3047	0,01307	0,00131	0,01219	0,8502	0,096	4,5	1,0	0,099

Table 3

Calibrated parameters, year 2015

ae	am	γe	γm	afl	afk	γfl	γfk	μ	αh	αg	Ai
1,99607	2,06427	0,750101	0,489141	6,21	13,97	0,3407	0,6593	47490966	6,043	1,05	0,9999

It\`s necessary to remark that all calculus parameters from table 3 have been recalculated according to the available information. SA matrices have been elaborated for years 2014, 2015, 2016. In the lack of data for year 2017, it was not possible to elaborate SA matrices. All needed data for year 2017 were obtained based on the existing statistical data for this year.

For years 2015-2016 exogenous variables were selected from SAM, coefficients of behavioral functions have been calibrated using SAM data and after that SOLVER application was applied to solve the model. Results of this calculus one can see in the Table 4. One can see that for year 2015 base year values and calculated values are the same. As for year 2017 there is a lack of data, macroeconomic indicators have been calculated using available information for this year, parameters, exogenous variables and some macroeconomic indicators were calculated on the base of known growth rate for Gross Domestic Product, then model coefficients were calibrated and model solved using SOLVER application. It is very hard to find equilibrium solution for year 2017 but after equilibrating initial data good solution has been obtained. For the future simulation calculus with changing taxes and subsidies policy can be effectuated. The purposes of this research deal with finding equilibrium solution and compare it with that known.

Table 4

Calculus results

Base year	Variables	2015	2016	2017	2018
0,10	CPI	0,10	1,10	1,096	1,10
1,000	P	1,000	1,099	1,099	1,099
0,8919	Pt	1,0000	1,3653	1,332723	1,3653
0,7429	Pd	0,7429	0,7429	0,742875	0,7429
1,1929	Pk	1,1929	1,1929	1,19286	1,1929
1,1097	Pl	1,1097	1,1097	1,109715	1,1097
0,9987	Pe	0,9974	0,9987	0,99869	0,9987
1,0131	Pm	1,0131	1,0131	1,013067	1,0131
109731921	B	109731921	71659468	77913625	72287084
41703652	GB	42587805,1	45394395,6	48681862	55607710
128639293	Y	128639293	112267405	120245209	128483861
27801811	S	27801811	30822387	32302952	42718034
15256337	Sh	15256337	20835210	22315775	23844750
12340517	GHtrf	12340517	352147	352147,4	352147
109731921	C	109731921	116547021	126718787	157681824
23095704	G	23095704	23391634	25587217	35052039
27801808	I	27801808	30506038	31971407	42279592
242862568	Xt	242862568	281180797	338415315	385464470
261552683	Xd	190400052	222332462	273486072	310334213
352081887	X	280929256	318385942	377432813	435315606
52462516	E	52462516	58848335	64929243	75130257
90529204	M	90529204	96053480	103946741	1249811393
52148008	L	52148008	56679888	60689328	60689328
27132075	K	27132075	30506038	34474402	42713054
81252459	Ls	666662217	58563029	62572469	62572469
27123075	BP	8791625	6042286	7854638	18688276

Conclusions. Small General Equilibrium Calculated model was adapted to the economic realities of the Republic of Moldova. Private, Public, Manufacture and Foreign sectors are examined. Optimization problems were formulated for all sectors. It discussed all aspects of building a CGE model such as the fundamental assumptions, the derivation of model equations, the estimation of parameters and model balancing. Using Lagrange multiplier method behavioral functions have been obtained. Social Accounting Matrices for years 2014-2016 were elaborated. Because of the nature of the SAM, the sums of columns are equal to the sums of lines, a lot of simulations can be done, and policy change can be evaluated. It is necessary to mark that SAM can be constructed with at least one year lag and, in consequence, economy evolution for the future can be effectuated on the previous year economy data. So, all parameters and functions must be carefully prepared. In order to effectuate simulations using presented General Equilibrium Calculated Model four sets of data are need: constant parameters which are find from scientific literature;

exogenous variables determined for the time period in examination; calibrated parameter for behavioral functions used in this study; and policy variables, part of the exogenous variables, such as tax rates, rates of the subsidies, social assistance rates, social contributions rates etc. For year 2017 exogenous and policy variables were obtained, behavioral coefficients have been calculated and model was solved using Solver tool. It is recommended to be very cautiously preparing all needed data for calculus; it will determine quality of results. Calculus effectuated for years 2015-2016 confirm that all data in SAM matrix were equilibrated. On their basis behavioral functions were calibrated. Calculus demonstrates that calibrated coefficients of these functions are very sensible to changes in statistical data and in consequence the equilibrium solutions of the examined model are hard to obtain. However, such a model can be used for simulation calculus changing tariff rates, rate of the subsidies and some exogenous variables. For the future it will be interesting to formulate multi-sector model involving intermediates goods and Input-Output matrix.

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